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Robust Control of Uncertain Multiple T-S Fuzzy Neutral Systems with Time-Varying Delays

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Abstract

This paper deals with the problem of robust control for a class of uncertain multiple Takagi-Sugeno fuzzy neutral systems with time-varying delays.A fuzzy filter is constructed,which ensures the robust stability.Linear matrix inequality approach is applied.

Keywords: robust control,T-S fuzzy neutral systems,time-varying delays

Problem Formulation

Consider the following class of T-S fuzzy neutral systems with time-varying delays:

Plant rule i : IF $s_1(t)$ is μ_{i1} ,and $s_2(t)$ is μ_{i2} ,and …and $s_g(t)$ is μ_{ig} ,THEN

$$\dot{x}(t)=[A_i+\Delta A_i(t)]x(t)+[A_{hi}+\Delta A_{hi}(t)]x(t-d_1(t))+[A_{di}+\Delta A_{di}(t)]\dot{x}(t-d_2(t)) \\ +[B_i+\Delta B_i(t)]u(t)+[B_{hi}+\Delta B_{hi}(t)]u(t-d_3(t)) \quad (1)$$

$$z(t)=[C_i+\Delta C_i(t)]x(t)+[C_{hi}+\Delta C_{hi}(t)]x(t-d_1(t))+[D_i+\Delta D_i(t)]u(t) \\ +[D_{hi}+\Delta D_{hi}(t)]u(t-d_3(t)) \quad (2)$$

$$x(t)=\phi(t)$$

$$t\in[-\max(d_1,d_2,d_3,d_4),0] \quad (3)$$

where μ_{ij} is the fuzzy set and r is the number of IF-THEN rules; $x(t)\in R^n$ is state vector, $u(t)\in R^s$ is input control , $d_i(t)$ are time-varying delays,which are assumed to satisfy $0\le d_i(t)\le h$, $\dot{d}_i(t)\le\mu$, h and μ are constants, $d_i=\sup_t d_i(t)$. $A_i, A_{hi}, A_{di}, B_i, B_{hi}(t)$ are known constant matrices with appropriate dimensions. $\Delta A_i(t), \Delta A_{hi}(t), \Delta A_{di}(t), \Delta B_i(t), \Delta B_{hi}(t)$ are unknown matrices representing time-varying parameter uncertainties.They are assumed to satisfy the following form:

$$[\,\Delta A_i(t)\ \ \Delta A_{hi}(t)\ \ \Delta A_{di}(t)\ \ \Delta B_i(t)\ \ \Delta B_{hi}(t)\,]=F_1\prod\nolimits_1(t)[\,E_{A_i}\ \ E_{A_{hii}}\ \ E_{A_{dii}}\ \ E_{B_i}\ \ E_{B_{hi}}\,]$$

$$[\,\Delta C_i(t)\ \ \Delta C_{hi}(t)\ \ \Delta D_i(t)\ \ \Delta D_{hi}(t)\,]=F_2\prod\nolimits_2(t)[\,E_{C_i}\ \ E_{C_{hii}}\ \ E_{D_i}\ \ E_{D_{hi}}\,]$$

And $\prod_1^T(t)\Pi_1(t)\le I$, $\prod_2^T(t)\Pi_2(t)\le I$.

By fuzzy blending,the dynamic fuzzy model in (1)-(4) can be represented by

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(s(t))\{[A_i+\Delta A_i(t)]x(t)+[A_{hi}+\Delta A_{hi}(t)]x(t-d_1(t))+[A_{di}+\Delta A_{di}(t)]\dot{x}(t-d_2(t)) \\ +[B_i+\Delta B_i(t)]u(t)+[B_{hi}+\Delta B_{hi}(t)]u(t-d_3(t))\}$$

$$z(t)=\sum_{i=1}^{r}h_i(s(t))\{[C_i+\Delta C_i(t)]x(t)+[C_{hi}+\Delta C_{hi}(t)]x(t-d_1(t))+[D_i+\Delta D_i(t)]u(t) \\ +[D_{hi}+\Delta D_{hi}(t)]u(t-d_3(t))\}$$

where $h_i(s(t)) = \dfrac{\varpi_i(s(t))}{\sum_{j=1}^{r}\varpi_j(s(t))}$, $\varpi_i(s(t)) = \prod_{j=1}^{g}\mu_{ij}(s_j(t))$, $s(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_g(t)\end{bmatrix}$,

in which $\mu_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in μ_{ij} .Then we can deduce that

$$\varpi_i(s(t)) \ge 0, \sum_{j=1}^{r}\varpi_j(s(t)) > 0,$$

for all t. Therefore,for all t, $h_i(s(t)) \ge 0, \sum_{i=1}^{r} h_i(s(t)) = 1$.

Main Result

Theorem For given scalars $h > 0$ and μ, if there exist $P = P^T > 0$, $R_i = R_i^T > 0$, $i = 1,2,3$, $S = S^T > 0$, $Z = Z^T > 0$, $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ * & X_{22} & X_{23} & X_{24} \\ * & * & X_{33} & X_{34} \\ * & * & * & X_{44}\end{bmatrix} \ge 0$, N_i and M_i $(i = 1,2,3)$ are matrices with appropriate dimensions, make the following LMI established,

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44}\end{bmatrix} < 0, \quad \psi = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & N_1 + M_1 \\ & X_{22} & X_{23} & X_{24} & N_2 \\ & & X_{33} & X_{34} & M_2 \\ & & & X_{44} & N_3 + M_3 + Z \\ & & & & 0 \end{bmatrix} \ge 0$$

$$\Xi_{11} = P\bar{A}_{ij}(t) + \bar{A}_{ij}(t)^T P + R_1 + R_3 + N_1 + N_1^T + M_1 + M_1^T + hX_{11} + \bar{A}_{ij}(t)^T (S + hZ)\bar{A}_{ij}(t)$$

$$\Xi_{12} = P[A_{hij} + \Delta A_{hij}(t)] - N_1 + N_2^T + A_{ij}(t)^T (S + hZ)[A_{hij} + \Delta A_{hij}(t)]$$

$$\Xi_{13} = P[B_{hij} + \Delta B_{hij}(t)]K_j - M_1 + M_2^T + hX_{13} + \bar{A}_{ij}(t)^T (S + hZ)[B_{hij} + \Delta B_{hij}(t)]K_j$$

$$\Xi_{14} = P[A_{dij} + \Delta A_{dij}(t)] + N_3^T + M_3^T + hX_{14} + \bar{A}_{ij}(t)^T (S + hZ)[A_{dij} + \Delta A_{dij}(t)]$$

$$\Xi_{22} = -(1-\mu)R_1 - N_2 - N_2^T + hX_{22} + [A_{hij} + \Delta A_{hij}(t)]^T (S + hZ)[A_{hij} + \Delta A_{hij}(t)]$$

$$\Xi_{23} = [A_{hij} + \Delta A_{hij}(t)]^T (S + hZ)[B_{hij} + hX_{23} + \Delta B_{hij}(t)]K_j$$

$$\Xi_{24} = N_3^T + hX_{24} + [A_{hij} + \Delta A_{hij}(t)]^T (S + hZ)[A_{dij} + \Delta A_{dij}(t)]$$

$$\Xi_{33} = -(1-\mu)R_3 - M_2 - M_2^T + hX_{33} + [B_{hij} + \Delta B_{hij}(t)]^T (S + hZ)[B_{hij} + \Delta B_{hij}(t)]$$

$$\Xi_{34} = -M_3 + hX_{34} + K_j^T [B_{hij} + \Delta B_{hij}(t)]^T (S + hZ)[A_{dij} + \Delta A_{dij}(t)]$$

$$\Xi_{44} = -(1-\mu)S + hX_{44} + [A_{dij} + \Delta A_{dijj}(t)]^T (S + hZ)[A_{dijj} + \Delta A_{dij}(t)]$$

then the state feedback control law is exist:

Control Rule i:IF $s_1(t)$ is μ_{i1} and $s_2(t)$ is μ_{i2} …and $s_g(t)$ is μ_{ig} ,THEN

$$u(t) = K_i x(t)$$

in which $K_i = Y_i X^{-1}$, $i = 1,2,\ldots,r$ are the control gain matrices, therefore the fuzzy state feedback controller can be given by the following: $u(t) = \sum_{i=1}^{r} h_i(s(t))K_i x(t)$ .So that the closed loop multiple system (1)-(4) is asymptotically stable.

Proof: Put $u(t) = \sum_{i=1}^{r} h_i(s(t))K_i x(t)$ into (1),we can get

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$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\{\overline{A}_{ij}(t)x(t)+[A_{hi}+\Delta A_{hi}(t)]x(t-d_1(t))$$
$$+[A_{di}+\Delta A_{di}(t)]\dot{x}(t-d_2(t))+[B_{hi}+\Delta B_{hi}(t)]K_j x(t-d_3(t))\}$$

where $\overline{A}_{ij}(t)=\overline{A}_{ij}+\Delta\overline{A}_{ij}(t)$, $\overline{A}_{ij}=A_i+B_iK_j$, $\Delta\overline{A}_{ij}(t)=\Delta A_i(t)+\Delta B_i(t)K_j$

Define the following *Lyapunov* functional candidate:

$$V(t)=V_0(t)+V_1(t)+V_2(t)$$

where $V_0(t)=x(t)^T Px(t)$

$$V_1(t)=\int_{t-d_1(t)}^{t}x(s)^T R_1 x(s)ds+\int_{t-d_3(t)}^{t}x(s)^T R_3 x(s)ds$$
$$V_2(t)=\int_{t-d_2(t)}^{t}\dot{x}(s)^T S\dot{x}(s)ds+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)^T Z\dot{x}(s)dsd\theta$$

then $\dot{V}_0(t)=2x(t)^T P\dot{x}(t)$

$$\dot{V}_1(t)=x(t)^T R_1 x(t)-(1-\dot{d}_1(t))x(t-d_1(t))^T R_1 x(t-d_1(t))+x(t)^T R_2 x(t)$$
$$-(1-\dot{d}_3(t))x(t-d_3(t))^T R_3 x(t-d_3(t))$$
$$\dot{V}_2(t)=\dot{x}(t)^T S\dot{x}(t)-(1-\dot{d}_2(t))\dot{x}(t-d_2(t))^T S\dot{x}(t-d_2(t))+h\dot{x}(t)^T Z\dot{x}(t)-\int_{t-h}^{t}\dot{x}(s)^T Zx(s)ds$$

so we can get

$$\dot{V}(t)\le 2x(t)^T P\dot{x}(t)+x(t)^T R_1 x(t)-x(t-d_1(t))^T R_1 x(t-d_1(t))+x(t)^T R_2 x(t)$$
$$-x(t-d_2(t))^T R_2 x(t-d_2(t))+\dot{x}(t)^T S\dot{x}(t)-\dot{x}(t-d_2(t))^T S\dot{x}(t-d_2(t))$$

And one side,

$$2x(t)^T P\dot{x}(t)$$
$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\{x(t)^T[P\overline{A}_{ij}(t)+\overline{A}_{ij}(t)^T P]x(t)+2x(t)^T P[A_{hij}+\Delta A_{hij}(t)]x(t-d_1(t))$$
$$+2x(t)^T P[A_{dij}+\Delta A_{dij}(t)]\dot{x}(t-d_2(t))+2x(t)^T P[B_{hij}+\Delta B_{hij}(t)]K_j x(t-d_3(t))\}$$

on the other hand, by Newton Leibniz formula,we can get he following formulas,

$$[x(t)^T N_1+x(t-d_1(t))^T N_2+\dot{x}(t-d_2(t))^T N_3]\cdot[x(t)-\int_{t-d_1(t)}^{t}\dot{x}(s)ds-x(t-d_1(t)]=0$$
$$[x(t)^T M_1+x(t-d_3(t))^T M_2+\dot{x}(t-d_2(t))^T M_3][x(t)-\int_{t-d_3(t)}^{t}\dot{x}(s)ds-x(t-d_3(t))]=0$$

So, the time derivative of $V(t)$ is computed as,

$$\dot{V}(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(t))h_j(s(t))\{\{x(t)^T[P\overline{A}_{ij}(t)+\overline{A}_{ij}(t)^T P]x(t)+2x(t)^T P[A_{hij}+\Delta A_{hij}(t)]x(t-d_1(t))$$
$$+2x(t)^T P[A_{dij}+\Delta A_{dij}(t)]\dot{x}(t-d_2(t))+2x(t)^T P[B_{hij}+\Delta B_{hij}(t)]K_j x(t-d_3(t))\}+x(t)^T(R_1+R_3)x(t)$$
$$-(1-\mu)x(t-d_1(t))^T R_1 x(t-d_1(t))-(1-\mu)x(t-d_3(t))^T R_3 x(t-d_3(t))+\dot{x}(t)^T(S+hZ)\dot{x}(t)$$
$$-(1-\mu)\dot{x}(t-d_2(t))^T S\dot{x}(t-d_2(t))-\int_{t-h}^{t}\dot{x}(s)^T Z\dot{x}(s)ds+2[x(t)^T N_1+x(t-d_1(t))^T N_2+\dot{x}(t-d_2(t))^T N_3]$$
$$\cdot[x(t)-\int_{t-d_1(t)}^{t}\dot{x}(s)ds-x(t-d_1(t)]+2[x(t)^T M_1+x(t-d_3(t))^T M_2+\dot{x}(t-d_2(t))^T M_3][x(t)$$
$$-\int_{t-d_3(t)}^{t}\dot{x}(s)ds-x(t-d_3(t))]+h\eta_1(t)^T X\eta_1(t)-\int_{t-d_1(t)}^{t}\eta_1(t)^T X\eta_1(t)ds-\int_{t-d_3(t)}^{t}\eta_1(t)^T X\eta_1(t)ds\}$$
$$=\eta_1(t)^T\Xi\eta_1(t)-\int_{t-d}^{t}\eta_2(t)^T\psi\eta_2(t)ds$$

where $\eta_1(t)=[x(t)^T \quad x(t-d_1(t))^T \quad x(t-d_3(t))^T \quad \dot{x}(t-d_2(t))]$

$$\eta_2(t) = [x(t)^T \quad x(t-d_1(t))^T \quad x(t-d_3(t))^T \quad \dot{x}(t-d_2(t)) \quad \dot{x}(s)]$$

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ * & X_{22} & X_{23} & X_{24} \\ * & * & X_{33} & X_{34} \\ * & * & * & X_{44} \end{bmatrix} \geq 0$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} < 0$$

Then we obtain $\dot{V}(t) < 0$, that is to say the closed loop multiple system is asymptotically stable.

References

[1] Ze Li,Shengyuan Xu,Yun Zou,Yuming Chu, Robust H_∞ filter design of uncertain T-S fuzzy neutral systems with time-varying delays, International Journal of Systems Science,2011,42:7, 1231-1238.

[2] Zhou,S.,Li,T.,Robust Stabilization for Delayed Discrete-time Fuzzy Systems via Basis-dependent Lyapunov-Kasovskii Function,Fuzzy Sets and Systems,2005,151,139-153.

[3] Zhang,B.,Zhou,S.,and Li T., A New Approach to Robust and Non-fragile H_∞ Control for Uncertain Fuzzy Systems,Information Sciences,2007,177,5118-5133.

[4] Zhang,H.,Lun,S.,and Liu,D.,Fuzzzy H_∞ Filter Design for a Class of Nonlinear Discrete-time Systems with Multiple Time Delays,IEEE Transactions on Fuzzy Systems,2007,15,453-469

[5] Yan Zhao,Huijun Gao,Member,IEEE,James Lam,Senior Member,IEEE,and Baozhu Du,Stability and Stabilization of Delayed T-S Fuzzy Systems:A Delay Partitioning Approach,IEEE Transactions on Fuzzy Systems,Vol.17,No.4,August 2009.

[6] Z.Wang,Y.Liu,X.Liu, H_∞ filtering for uncertain stochastic time-delay systems with sector-bounded nonlinearities.Automatica 44(5),1268-1277(2008).

[7] Jiyao An,Guilin Wen,Improved stabillity criteria for time-varying delayed T-S fuzzy systems via delay partitioning approach,Fuaay Sets and Systems,2011,185:83-94.

[8] Zhicheng Li,Hongyang Liu,Huijun Gao,A delay partitioning Approach to H_∞ filtering for continuous time-delay systems,Circuits System Signal Process(2011)30:501-513.